Diskussion

Wie man Tab. 1 entnimmt, befinden sich alle elf von uns gemessenen Häufigkeitsverhältnisse innerhalb der kombinierten Fehlergrenzen in Übereinstimmung mit NIERS Werten. Zugleich ist aber auch an der letzten Spalte von Tab. 1 ersichtlich, daß wir zwar innerhalb der Fehlergrenze, aber dennoch ziemlich systematisch eine Anhebung der Häufigkeiten der leichten Isotope gegenüber den Werten von NIER finden. Dies bewirkt, daß sich die prozentuale Verteilung der Isotope (siehe Tab. 2) zwar bei den beiden Meßreihen in befriedigender Übereinstimmung befindet, jedoch für den ^{136}Xe-Anteil etwas auseinander fällt.

Obgleich die Abweichungen klein sind, nehmen wir wegen ihrer relativ geringen Streuung an, daß sie durch einen systematischen Fehler verursacht sind. Wir haben daher zunächst unsere Apparatur auf entsprechende Fehlerquellen hin untersucht (insbesondere Gaseinlaß-System, Spannungsteiler, Nachweiselektronik), haben aber keine Ursache für die beobachteten Abweichungen finden können. Leider muß aber wegen des Fehlens geeigneter Eichproben die wichtige Annahme ungeprüft bleiben, daß das Analysierfeld eine massenunabhängige Transmission hat. Setzen wir die NIERschen Werte als richtig voraus, können wir nur schließen, daß das verwendete Massenfilter auch bei einem Auflösungsvermögen von ca. 400 eine erfreulich geringe Massendiskriminierung zeigt.

Andererseits zeigt eine kritische Durchsicht der von NIER verwandten Methode folgendes:

Die Bestimmung der bei den Xenon-Messungen auftretenden Massendiskriminierung erfolgte durch eine zweistufige Eichmessung am Verhältnis ^{128}Xe/^{136}Xe. Zunächst wurde bei der Messung von $(^{128}\mathrm{Xe}/^{136}\mathrm{Xe})^{3+}$ $D=0{,}15$ ermittelt, im zweiten Schritt mit einfach ionisierten Ionen $(^{128}\mathrm{Xe}/^{136}\mathrm{Xe})^{1+}$ $D=0{,}015$. Der letztere Wert ist allerdings so klein, daß sein Einfluß auf die Meßwerte fast zu vernachlässigen ist. Im Lichte der eingangs erwähnten starken Variabilität dieser Diskriminierungsfaktoren scheint es uns daher nicht völlig von der Hand zu weisen zu sein, daß insbesondere der zweite Faktor etwas zu klein ermittelt wurde.

Spallation Formula Deduced from Cascade and Evaporation Theories Compared with Experiments

U. SCHWARZ * and H. OESCHGER

Physikalisches Institut, University of Bern, Switzerland

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A semiempirical formula $\sigma(\Delta A, E_0, A_0)$, giving the sum of the spallation cross sections of the products of an isobar as a function of the mass loss ΔA, the primary energy E_0 of the irradiating particle and the atomic mass number A_0 of the target nucleus is deduced. The formula is intended to be used for the interpretation of the spallation product distribution in meteorites, in the atmosphere of the earth and hopefully in cosmic dust and lunar surface samples.

Due to the refinement of observational techniques considerable experimental data are now available concerning production rates of spallation products (induced by high energy particles) in meteorites and the atmosphere. The interpretation of the results is very often difficult because most of the (differential) cross sections are not determined experimentally. It is therefore necessary to make estimates of the cross sections of the products, depending on the primary energy E_0 of the irradiating particle and the atomic mass number A_0 of the target element, covering a wide range of the parameters. A special problem arises in meteorites, where the irradiation time is so great, that the spallation products of stable nuclei are the accumulated products of the whole isobar. In this report an attempt is made to establish a relation $\sigma(\Delta A, E_0, A_0)$, giving the sum of the cross sections of the products of an isobar, corresponding to a mass loss ΔA. RUDSTAM [1, 2] and HONDA et al. [3] found that it was possible to give such a relationship

* Now at Kapteyn Laboratory, Groningen, The Netherlands.

1 G. RUDSTAM, University of Uppsala, Thesis 1956.

2 G. RUDSTAM, E. BRUNINX, and A. C. PAPPAS, Phys. Rev. **126**, 1852 [1962]. — G. RUDSTAM, 1966, in press.

3 M. HONDA and D. LAL, Phys. Rev. **118**, 1618 [1960].

an analytic form for a fixed A_0; namely that of a Cu target. The relation is here extended for other target masses, using the results of Monte Carlo calculations (METROPOLIS et al.[4], DOSTROVSKY et al.[5]); this relation has been reported already by GFELLER et al.[6]. The analytic representation of spallation products has the advantage that it allows the derivation of simple relations among the production rates of spallation products, depending on ΔA and the energy spectrum of the reacting particles, which in special cases nearly equals the energy spectrum of cosmic radiation[7].

The cross section for the production of the individual nuclei on the isobar can be derived from our relation $\sigma(\Delta A, E_0, A_0)$, making use of the experimental result that these cross sections can be approximated by a GAUSSIAN function[1, 2, 8].

In the Monte Carlo calculations, the statistical fluctuations are high for the individual cross sections of the products due to the small probability for their production, especially for large mass or high primary energies. Therefore only the mean mass losses for different target elements with mass A_0 and primary energy E_0 will be determined and compared with the empirically determined mass losses.

1. Experimental Mean Mass Losses

In order to simplify the determination of the empirical mass losses, the spectrum of the sum of all products of an isobar is assumed exponential, as given by the empirical formula of RUDSTAM[1]

$$\sigma(\Delta A) = \text{const} \cdot e^{-pA}. \tag{1}$$

For normalisation it is assumed, that (1) is valid in the range $1 \leqq \Delta A \leqq A_0$. The mean mass loss per interaction is then:

$$\begin{aligned} \overline{\Delta A} &= \left[\sum_{\Delta A=1}^{A_0} \sigma(\Delta A)\, \Delta A \right] \Big/ \sum_{\Delta A=1}^{A_0} \sigma(\Delta A) \\ &\approx \int_1^\infty \sigma(\Delta A)\, \Delta A \, \mathrm{d}(\Delta A) \Big/ \int_1^\infty \sigma(\Delta A)\, \mathrm{d}(\Delta A) \qquad (2) \\ &= 1 + 1/p\,. \end{aligned}$$

Accurate determinations of p are only available for the target element Cu. Values of $\overline{\Delta A}$ of this target element at different primary energies are given in the table.

2. Mean Mass Losses by the Monte Carlo Calculations

From the papers of METROPOLIS et al.[4] and DOSTROWSKY et al.[5] mean mass losses per interaction can be calculated in a first approximation by:

$$\overline{\Delta A} = \overline{\Delta A'} + \overline{\Delta A''} \tag{3}$$

where $\overline{\Delta A'}$ is the mass loss due to the primary cascade and $\overline{\Delta A''}$ the mass loss due to the evaporation of the exited nucleus. There is a pronounced dependence on the primary energy E_0 and the target mass A_0, which arises mainly from the number of primary cascade particles and the deposition of excitation energy in the remnant nucleus, respectively.

The values of $\overline{\Delta A}$ according to equation (3) for the target element Cu at different primary energies are listed. The comparison with the experimentally determined mean mass losses from eq. (2) shows a slight overestimation by the Monte Carlo calculations. The fourth column of the table gives the ratio $\overline{\Delta A}$ (Monte Carlo) to $\overline{\Delta A}$ (experiment), which seems to be fairly constant; the average of the listed values is 1.17. In the Monte Carlo calculations $\overline{\Delta A'}$ and, therefore $\overline{\Delta A}$ depends critically on the constant a_0 of the adopted nuclear radius $a_0\, A_0^{1/3}$. The agreement between $\overline{\Delta A}$ (Monte Carlo) and $\overline{\Delta A}$ (experimental) can be considered as very good. In the following all ΔA values predicted by the Monte Carlo calculations will be corrected by the factor 1.17.

E_0 (MeV)	$\overline{\Delta A}$ Monte Carlo	$\overline{\Delta A}$ experim.[9]	ratio
100	3.4	3.0	1.13
200	4.6	4.1	1.12
500	8.1	6.9	1.18
1000	12.9	10.5	1.26
1840	19	17	1.1

Table 1.

3. Derivation of the Parameters of the Spallation Product Distribution

Using the $\overline{\Delta A}$ predicted by the Monte Carlo calculations p can be determined for different target

[4] N. METROPOLIS, R. BIVINS, M. STORM, A. TURKEVICH, J. M. MILLER, and G. FRIEDLANDER, Phys. Rev. **110**, 185 [1958].

[5] I. DOSTROVSKY, P. RABINEWITZ, and R. BIVINS, Phys. Rev. **111**, 1659 [1958].

[6] CHR. GFELLER, F. G. HOUTERMANS, H. OESCHGER, and U. SCHWARZ, Helv. Phys. Acta **34**, 466 [1961].

[7] J. GEISS, H. OESCHGER, and U. SCHWARZ, Space Sci. Rev. **1**, 197 [1962].

[8] D. W. BARR, University of California Radiation Laboratory, Report UCRL-3793 [1957].

[9] Interpolated values at the given energies, based upon the experiment results on RUDSTAM[1] and BARR[8].

elements and primary energies from the formula (2) and (3) after correction by the factor 1.17. In the figure the p's are plotted and straight lines are drawn. It can be seen that the energy dependence of p for different target elements is similar and the slopes of the lines show only differences which seem

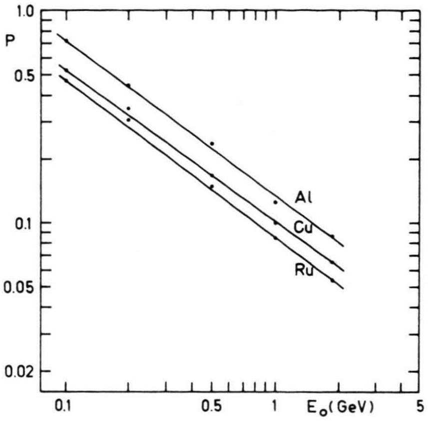

Fig. 1.

insignificant. Therefore, the same energy dependence is assumed and p is calculated using the following interpolation formula

$$p \approx c_1 E_0^{-2/3} \frac{1}{1+c_2 A_0}. \tag{4}$$

The numerical values of the constants are:

$$c_1 = 0.237\ (\mathrm{GeV}^{2/3}), \quad c_2 = 0.020$$

when normalized on Cu. This gives a good fit for Al and a somewhat poorer one (10%) for Ru.

In order to determine the constant in (1), the fact can be used that the sum of all cross sections of the various products must equal the total inelastic cross section. $\sigma_0 A_0^{2/3}(1-t)$, where σ_0 is a constant and t the transparency. The sum of all cross section, according to l and approximated by an integral, gives:

$$\sigma_0 A_0^{2/3}(1-t) \approx \int_1^\infty \mathrm{const}\cdot e^{-p\Delta A}\, \mathrm{d}(\Delta A) = \mathrm{const}\, \frac{1}{p} e^{-p}. \tag{5}$$

Using the values of the transparencies as given by the Monte Carlo calculations one finds that the ratio of $e^{-p}/(1-t)$ is close to one in an energy range above 300 MeV. Therefore one may write

$$\sigma(\Delta A, E_0, A_0) = \sigma_0 A_0^{2/3} p\, e^{-p\Delta A} \tag{6}$$

with p as given in (4). The constant $\sigma_0 = 60$ mb when normalized to the results of Cu at 5.7 GeV [8]. Comparisons with experiment shows that the formula (6) is valid for Cu in an energy range from 100 MeV up to at least 5.7 GeV [1, 8]. At very high energies, the energy dependence seems to fail. Empirical values of p at 24 GeV [2] indicate a change in slope of the p versus energy curve towards an asymptotic constant value. The A_0 dependence has not yet been conclusively verified.

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